Separable Equations Worksheet

1. $\frac{dy}{dx} = \frac{2x}{y}$

$$\int y\,dy = \int 2x\,dx$$
$$\tfrac{1}{2}y^2 = x^2 + C$$
$$y^2 = 2x^2 + C$$
$$y = \pm\sqrt{2x^2 + C}$$

2. $y\frac{dy}{dx} - e^x = 0$

$$y\frac{dy}{dx} = e^x$$
$$\int y\,dy = \int e^x\,dx$$
$$\tfrac{1}{2}y^2 = e^x + C$$
$$y^2 = e^x + C$$
$$y = \pm\sqrt{e^x + C}$$

3. $yy' + 5x^2 = x$

$$y\frac{dy}{dx} = x - 5x^2$$
$$\int y\,dy = \int -5x^2 + x\,dx$$
$$\tfrac{1}{2}y^2 = \tfrac{-5}{3}x^3 + \tfrac{1}{2}x^2 + C$$
$$y^2 = \tfrac{-10}{3}x^3 + x^2 + C$$
$$y = \pm\sqrt{\tfrac{-10}{3}x^3 + x^2 + C}$$

4. $\frac{dy}{dx}(5-2x^2) = 3xy$

$(5-2x^2)\,dy = 3xy\,dx$

$\int \frac{1}{y}\,dx = \int \frac{3x}{5-2x^2}\,dx$

$\ln|y|$ $\quad u = 5-2x^2 \quad \int \frac{3x}{u} \cdot \frac{du}{-4x}$

$\frac{du}{dx} = -4x \quad -\frac{3}{4}\int \frac{1}{u}\,du$

$dx = \frac{du}{-4x} \quad -\frac{3}{4}\ln|5-2x^2| + C$

$\ln|y| = -\frac{3}{4}\ln|5-2x^2| + C$

$y = \pm e^{-\frac{3}{4}\ln|5-2x^2| + C}$

5. $y\frac{dy}{dx} = 3x+2 \qquad f(5)=7$

$\int y\,dy = \int 3x+2\,dx$

$\frac{1}{2}y^2 = \frac{3}{2}x^2 + 2x + C$

$\frac{1}{2}(7)^2 = \frac{3}{2}(5)^2 + 2(5) + C$

$\frac{49}{2} = \frac{75}{2} + \frac{20}{2} + C$

$\frac{49}{2} = \frac{95}{2} + C$

$-23 = C$

$\frac{1}{2}y^2 = \frac{3}{2}x^2 + 2x - 23$

$y^2 = 3x^2 + x - 46$

$y = \pm\sqrt{3x^2 + x - 46}$

$y = \sqrt{3x^2 + x - 46}$

6. $\frac{dr}{ds} = e^{r+s}$ $\quad r(1)=0$

$\frac{dr}{ds} = e^{r} \cdot e^{s}$

$dr = e^{r} \cdot e^{s} \, ds$

$\frac{1}{e^{r}} dr = e^{s} ds$

$\int e^{-r} dr = \int e^{s} ds$

$= -r \quad \int e^{u} \cdot \frac{du}{-1}$ $\quad e^{s} + C$

$\frac{du}{dr} = -1 \quad -\int e^{u} du$

$du = -dr \quad -e^{-r}$

$-e^{-r} = e^{s} + C$

$-e^{-(0)} = -e^{(1)} + C$

$-1 = -e + C$

$C = e - 1$

$-e^{-r} = e^{s} + e - 1$

$e^{-r} = -e^{s} - e + 1$

$-r = \ln(-e^{s} - e + 1)$

$r = -\ln(-e^{s} - e + 1)$

7. $\sqrt{x} + \sqrt{y}\, y' = 0$ $\quad y(1) = 4$

$y^{1/2} \frac{dy}{dx} = -x^{1/2}$

$\int y^{1/2} dy = \int -x^{1/2} dx$

$\frac{2}{3} y^{3/2} = -\frac{2}{3} x^{3/2} + C$

$\frac{2}{3}(4)^{3/2} = -\frac{2}{3}(1)^{3/2} + C$

$\frac{16}{3} = -\frac{2}{3} + C$

$C = 6$

$3 \cdot \left(\frac{2}{3} y^{3/2} = -\frac{2}{3} x^{3/2} + 6 \right)$

$2y^{3/2} = -2x^{3/2} + 18$

$y^{2/3} = -x^{3/2} + 9$

$y = (-x^{3/2} + 9)^{3/2}$

8. $y(1+x^2)y' - x(1+y^2) = 0$ $\qquad y(0) = \sqrt{3}$

$$y(1+x^2)\frac{dy}{dx} = x(1+y^2)$$

$$y(1+x^2)\,dy = x(1+y^2)\,dx$$

$$\int \frac{y}{1+y^2}\,dy = \int \frac{x}{1+x^2}\,dx$$

$u = 1+y^2$ $\qquad u = 1+x^2$

$\frac{du}{dy} = 2y$ $\qquad \frac{du}{dx} = 2x$

$\int \frac{\cancel{y}}{u}\cdot\frac{du}{2\cancel{y}}$ $\qquad \int \frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}}$

$\frac{1}{2}\int \frac{1}{u}\,du$ $\qquad \frac{1}{2}\int \frac{1}{u}\,du$

$$\frac{1}{2}\ln|1+y^2| = \frac{1}{2}\ln|1+x^2| + C$$

$$\frac{1}{2}\ln|1+(\sqrt{3})^2| = \frac{1}{2}\ln|1+(0)^2| + C$$

$$\frac{1}{2}\ln 4 = \frac{1}{2}\ln 1 + C$$

$$\ln 2 = 0 + C$$

$$2\cdot\left(\frac{1}{2}\ln|1+y^2| = \frac{1}{2}\ln|1+x^2| + \ln 2\right)$$

$$\ln|1+y^2| = \ln|1+x^2| + 2\ln 2$$

$$\cancel{\ln}|1+y^2| = \cancel{\ln}\,4|1+x^2|$$

$$|1+y^2| = 4|1+x^2|$$